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An Integrated Model of Perennial and Annual Crop Production for Sub-Saharan Countries

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This microeconomic model of household choice reflects the fact that crop production in Sub-Saharan Africa is dominated by smallholders who allocate household labor across annual and perennial crops and, in some cases, to wage labor markets.

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Crop production in Sub-Saharan Africa is dominated by smallholders who allocate household labor across annual and perennial crops and, in some cases, to wage labor markets.

Weaver has developed a microeconomic model of household choice which is consistent with observed characteristics of Sub-Saharan agricultural systems in terms of:

- Integrated production of annual and perennial crops (since households often produce both annuals and perennials). This interaction has been ignored in past models.

- Price uncertainty in markets that may be affected by government intervention.

- The potential for off-farm employment.

- Household consumption of crops produced on the farm (on-farm inventories are usually nonexistent).

- Household consumption of non-food goods, school fees, and so on.

Weaver considers variations is the model to establish their implications. These variations include differential buying and selling prices, fixed subsistence consumption constraints, participation in wage market labor, smuggling in response to goverment price control, and parallel markets with penalties for smuggling.

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I. INTRODUCTION

The objectives of this paper are to enumerate a general set of key structural elements of smallholder rainfeed production systems in Sub-Saharan Africa, to present a theoretical model of household choice which is consistent with the observed agricultural systems and involves integration of annual and perennial crop activities, and to discuss the empirical implementation of such a model. The determinants of annual and perennial crop supply have been studied extensively, though mostly within the context of an _ad hoc_ partial adjustment model. The literature has largely concentrated on crop-specific models and the interdependence of annual and perennial crops in production systems has been ignored. In Sub-Saharan Africa, smallholders typically allocate resources across all crop types over the growing season to achieve objectives of cash income as well as to provide subsistence food.

Of particular interest in the model developed, therefore, is consideration of the implications of (i) jointness in production technology and (ii) common production constraints for the specification and interpretation of empirical models of crop production. The model incorporates household labor allocation across wage market opportunities and production of annual food crops and perennial cash crops such as coffee, tea, or cocoa. While many of the markets for both cash crops and food crops are regulated by governments, local markets for food often provide important sources of cash income with prices that are unregulated. Therefore, a general specification is developed which allows analysis of the effects of alternative price policies.

The format of the paper is as follows. Section II reviews the key characteristics of observed agricultural systems in these countries and concludes by enumerating a set of key structural elements. An integrated theoretical model of smallholder production decisions is presented in Section III. The final section considers the empirical implementation of the theoretical model presented as well as its implications for the interpretation of existing empirical results for either annuals or perennials.

II. CHARACTERISTICS OF SMALLHOLDER AGRICULTURE IN SUB-SAHARAN AFRICA

Agricultural production results from a complex series of activities which allocate resources conditionally upon producer response to the economic environment. Profiles of the structural elements of the economic environment in Sub-Saharan Africa are limited and most often are found in very specific agricultural systems studies (e.g., Norman, 1980), in farm management case studies (e.g., Collinson, 1968), in anthropological descriptions of broader social, political and economic systems, or more generally in the context of development project appraisals. In addition, a variety of farm surveys provide important background information.

Although heterogeneity exists over a vast set of dimensions across Sub-Saharan Africa, the objectives of this paper can be met by adoption of a limited set of characteristics. A theoretical model is presented which is consistent with this general case, yet allows accommodation of special characteristics of particular applications. The following salient characteristics emerge from a synthesis of evidence concerning smallholder traditional systems.

Land Tenure. Bush-fallow systems for annual crops dominate much of Sub-Saharan rainfed agriculture. Perennials are most often grown on communal land where rights are held as a result of continuity of use. In the presence of population pressure, fallowing periods have been shortened, but the system has otherwise persisted. This observation can also be generalized to flood irrigated areas such as alluvial plains or bottom-land plots. Within the bush-fallow system, plots are chosen each year depending upon their appropriateness and state of readiness for specific crops. It is not unusual for a smallholder to cultivate a set of plots on varying slopes, altitudes, and soils in order to achieve desired growing conditions for particular crops. Plots are identified through field survey, cleared and prepared for planting. Initial planting is followed by a series of crops which exploit changing growing conditions as the potential changes through use.

Scale of Operation. Land area cultivated is typically less than six acres and may be influenced or constrained by availability of land, family size and subsistence requirements, labor supply, availability of seeds, and expected growing conditions. In many cases, land area is not constrained in the medium term, and, subject to the prerequisite of clearing, may not be constrained for a growing season.

Diversification of Crops. Four crop characteristics can be distinguished which present bases for diversification: (i) necessity of timely harvest; (ii) need for field surveillance; (iii) usefulness for food subsistence; and (iv) time required for maturity. The matrix in Table 1 presents examples of crop types. Diversification allows for improved labor capacity utilization, improved income flow and level, diversification of diet, and reduction of subsistence or cash income risk exposure.

Table 1: Smallholder Crop Characteristics

Crop Characteristics	Cases	Examples
Timeliness of Harvest	Timely:	Wheat, teff, sorghum
	Periodic:	Coffee, cocoa
	On-Demand:	Cassava, sweet potato
Field Surveillance	Limited:	Wheat, coffee, cocoa
	Regular:	Maize, sorghum, groundnuts
Food Subsistence	None:	Coffee, cotton, tea
	Typical:	Maize, sorghum, groundnuts, fruit
Time for Maturity	Short:	Vegetables
	Medium:	Cereals
	Long-term:	Coffee, cocoa, tea, fruit

Use of Inputs. The typical smallholder is a traditional farmer using only land, labor, seeds, and simple tools ranging from a planting stick to hoes and knives. In some cases, new variety seeds may be available though typically seeds are reserved out of the previous year's harvest. Seed selection is not practiced. Pesticides and insecticides are not typically used by smallholders and their supply is characterized by uncertain quality, price and quantity availability. Use of fertilizers by smallholders does occur, though their use is affected by similar supply problems.

Wage Markets. The existence of wage markets for labor varies, though typically opportunity exists in agricultural activities, wood gathering, or small rural industries. A critical issue relevant to modeling of production decisions is the existence of economically-feasible, off-farm employment opportunities. That is, although wage markets may exist, they may not be accessible to smallholders. Further, on- and off-farm employment may not be perfect substitutes in the household's preference function. For either of these reasons, wage markets may not represent effective alternative uses for household labor. Nonetheless, labor markets typically exist not only on a local basis, but importantly on a regional basis, often involving migration to neighboring countries. The model developed below initially assumes the wage market is absent and explores the implications of a wage market in the context of variations in model specification.

Agricultural Markets. Local village markets and larger regional weekly markets present an opportunity for farmers to sell or purchase all crops. Food crops are typically brought to market by a farm wife. Merchant women arbitrage food products across village and regional markets. Food purchases are made from the same markets. Inventories are managed at the village and regional levels by shopkeepers who assemble inventory through trading with farm wives. Seasonal variation in prices is significant and accompanies seasonal patterns of coincident farm sales and purchases. This absence of farm inventories results in a large differential between the average prices paid and received by a farm household for farm products.

The role of the traditional rural agricultural market for perennial cash crops varies depending on the existence and extent of government regulation. In some countries, crops are sold to government dealers in the village markets or at central depots. In other cases, these crops are assembled at rural markets or collected directly from farmers by specialized itinerant traders.

Household Consumption. Large differentials between buying and selling prices of food products, uncertain market availability, high market transaction costs, and pride in self-sufficiency are among a variety of factors which have been argued to motivate smallholders in their attempt to supply the household's food consumption needs. Crops are marketed to satisfy the demand for cash required for the purchase of non-food consumption goods, school fees, etc. However, the subsistence orientation of smallholders should not be interpreted as indicating that they aim to produce only sufficient to satisfy a fixed consumption target, and market the surplus. Instead, food preferences exist which are sensitive to both quantity and quality, or type, of food. Food consumption decisions are influenced rather than being completely determined by fertility and family characteristics.

The Role of Non-Food Cash Crops. The post-colonial role of non-food cash crops is not well-understood. In the presence of complete and efficient markets, welfare would be maximized by production of crops which yield the highest cash return to fixed resources. Cash could then be used to satisfy food and other demand. Nonetheless, in the absence of such rural markets, strong household incentives may exist for production of, and choice among, food crops to satisfy subsistence needs. Non-food cash crops, particularly perennials, may represent important opportunities for productive use of periodic slack resources and for the anticipated employment of family labor resources.

<u>Technological and Economic Interdependence between Annuals and Perennials</u>. Although cases are observed of specialization in annual, or in perennial crops, these crop types are often interdependent. Annuals and perennials are often intercropped, particularly during immature phases of perennial development. Such interdependence implies that weed control, moisture control, climatic exposure, and soil conditioning benefits may affect the perennial due to the annual crop's presence. However, even in the absence of such interdependence, the productivity of labor and other resources committed to a perennial may be conditional on the resources committed to annual crops. Annual crop residues provide valuable mulch material, and annual cropping may enhance pollination and affect pest and weed exposure to the perennial. Economically, the crops may be interdependent due to their joint utilization of fixed resources.

<u>The Roles of Animal and Crop Production</u>. Throughout Sub-Saharan Africa, livestock is produced wherever possible. Important constraints on feasibility result from disease exposure and water and grazing resources. Both goats and cattle are raised, tended typically by children while the animals graze on communal land that is not cropped. Because the animals are typically not enclosed, manure is not retrieved for use on crops. Although important exceptions exist to this situation in Sub-Saharan Africa, the model developed in this paper will assume livestock production is separable from crop production.

<u>On-Farm Inventories</u>. Physical and financial capacity to manage harvested output on-farm is often lacking in Sub-Saharan Africa. Although food

consumption plans are made, harvests may fall short and deficiency in home supplies must be made up through communal borrowing or sharing. Where inventories exist they are held intra-seasonally. Non-food perennials may play a role in providing cash for food purchases during periods of deficient home production; however, the likely coincidence of shortages in annual and perennial harvests implies that deficient households would face high food prices and market shortages, as well as low perennial crop prices induced by local increases in market supplies.

III. AN INTEGRATED MODEL

Past models of crop production have acknowledged the interdependence among annual crops, (see e.g. Weaver (1983)) while models of perennials have assumed that annual and perennial crop production is separable--leaving decisions separable with regard to resource allocation across the alternatives. However, even in the presence of non-joint production technologies the existence of limited resources such as labor would imply allocation decisions are non-separable. The objective of this section is to develop a theoretical model consistent with observed integrated production of annuals and perennials which can provide the motivation for an empirical approach capable of: (i) testing for the existence of non-separability of the crop types; and (ii) establishing economic interdependence of the crops. In so doing, a model which is consistent with the salient characteristics of smallholder production outlined above will be presented, conditions sufficient for separability will be established, and the comparative-statics of the non-separable case investigated.

Wickens and Greenfield (1973), Dowling (1979), French and Matthews, (1971), Hartley et al., (1985) and Trivedi (1986) have presented a variety of approaches to modeling a single perennial crop's production. Building on this literature, the present section will introduce a generalized perennial model integrated with annual crop production decisions. To proceed, we recognize the following essential conceptual elements of a perennial model.

(a) Short-term allocation decisions cannot usefully be distinguished from long-term decisions; instead, continual revision of intertemporal plans must be assumed.

(b) Capital accumulation must be described.

(c) Uncertainty in the economic environment must be acknowledged.

The following model is proposed:

A. Production of Perennial Crops

$$(1) \quad x(t,v) = \begin{cases} J\left(L^H(t,v);\ x^o(t,v)\right) & t-v > V \\ 0 & t-v < V \end{cases}$$

where t indicates the current time period,

v indicates the vintage of the perennial stock,

x^o is a vector of potential output,

L^H is a vector of harvest period inputs, for now restricted to labor,

x is a vector of perennial output achieved,

V represents the age of productive maturity, and

$J(.)$ represents a continuously differentiable function, non-decreasing in L^H and x^o, and concave

(2) $x^o(t,v) \equiv d\left(L^K(t,v);K(t,v)\right) + x^o(t-1,v)$

where L^K is a vector of inputs, for now labor, applied to improvement of potential output, e.g., renewal pruning or spraying;

$K(t,v)$ is a vector of the capital stock of perennials, e.g., the number of trees or bushes;

$d(.)$ represents a vector function indicating adjustments to productive potential, non-decreasing in L^k and K, and concave.

(3) $X(t) \equiv \sum_v x\ (t,v)$

The specification of production technology presented in (1)-(3) must be interpreted as reflecting a single biological potential, or plant type. Where alternative plant types (e.g., traditional or hybrids) are relevant, each plant type stock would be distinguished in (2).

B. Capital Accumulation of Perennial Stocks

(4) $K(t,v) = K(t-1,v) + I(L_t^I;\ t,v) - R\ (L_t^R;\ t,v) - D(e_t;\ t,v)$

where $I(.)$ represents a vector of the stock from new plantings, for each perennial crop,

L_t^I	labor used for $I(.)$,
$R(.)$	removals of stock,

L_t^R	labor used for R(.),
D(.)	exogenous changes in stocks,
e_t	exogenous factors,
K(t,v)	represents the vector of the stock of perennials of vintage v,
I(.), R(.), and D(.)	are each non-decreasing in L, and concave.

By redefinition, further inputs could be introduced.

C. Annual Crop Production and Technological Interaction with Perennials

(5) $G[x\ (t,v),\ Y(t),\ L^Y(t)] = 0$

where Y(t) is a vector of home-produced annual crops.

and G(.) is non-increasing in X and Y, non-decreasing in L^Y, and concave.

D. Labor Constraint

$$L_t^Y + L_t^I + L_t^R + L_t^H + L_t^K + S_t \leq \bar{L}_t(\theta)$$

(6) $\sum_{i=1}^{n} L_t^i + S_t \leq \bar{L}_t\ (\Theta)$

where Θ is a vector of demographic factors,

$\bar{L}_t(.)$ is total labor available,

L_t^i is labor of type i, and

S is total family leisure.

E. Income-Expenditure Constraint

$$(7)\quad P_X(t)'X(t) + P_Y(t)'\left[Y(t) - Q(t)\right] - P_C'C(t) = 0$$

Q(t) is a vector of consumption of annual crops, either home-produced or purchased in the market,

P_X (t) is a vector of selling prices of the perennials,

$P_Y(t)$ is a vector of purchasing/selling prices of Y(t), home-produced annual food crops,

C(t) is a vector of non-food consumer goods available at $P_C(t)$.

In further generalization, (i) C(t) might be substitutes for Z goods as defined by Hymer and Resnick (1969), (ii) different purchase and selling prices could be introduced, and (iii) fixed subsistence constraints could be introduced where consumption of crops are not preference-based.

F. Household Objectives

For now, we assume the absence of a wage labor market, and specify:

(8) $U = U\{S(t), Q(t), C(t)\}$

where U(.) is a well-behaved household utility function with $U' > 0$, $U'' < 0$

G. Household Choice Problem

(9) $\max \; W(t) = \sum_{t=1}^{\infty} \{1+r(t)\}^{-1} U\{S(t), Q(t), C(t)\}$

subject to (1) - (7).

In order to introduce price uncertainty, we reinterpret all prices as price expectations and assume risk neutrality. The existence of risk aversion is an important empirical issue for which little evidence exists. For the present purposes, hypothesis of risk neutrality will be maintained. To simplify for illustrative purposes, we assume only one perennial crop and one annual crop. Given continuity of (1) and (5), we may write:

$$G[J\{L^H(.);x^o\}, Y(t), L^Y(t)] = 0$$

as

$$(10) \quad Y(t) = F\left(L^H(t),\ L^Y(t);\ x^o(t,v)\right)$$

where $F(.) = G^{-1}(.)$

By substitution into (9), we have the following Lagrangian (where we suppress vintages):

$$(11) \quad \Psi(t) = \sum_{t=1}^{\infty} \delta(t)\ U\left(\bar{L}(t,\theta)-\Sigma L^i(t),\ Q(t),C(t)\right)$$

$$+ \lambda_t \Big\{P_X(t)'J\left(L^H(t);\ x^o(t)\right) + P_Y(t)'[\left(F(L^H(t),\ L^Y(t);\ x^o(t,v)\right)$$

$$-Q(t)\big)] - P_C(t)'C(t)\Big\}$$

where

$$\delta(t) = \left(1-r(t)\right)^{-1}$$

$$x^o(t) = d\left(L^K(t),\ K(t)\right) + x^o(t-1)$$

$$K(t) = K(t-1) + I\left(L^I(t);\ t\right) - R\left(L^R(t);\ t\right) - D(e_t;t)$$

H. Interpretation of the Household Choice Problem

As stated, the household faces a dynamic optimization problem. Production possibilities could be made stochastic in a variety of ways. For example, shifts in the effective perennial stock might induce changes in potential production; and/or stochastic changes could induce changes in annual crop productivity. For now, the specification can be interpreted as consistent with static expectations of productivity which leads to a fixed production possibilities frontier. Future prices have been assumed to be stochastic and risk neutral choice is reflected in the interpretation of all future prices as expected prices.

No structure to the evolution of future expected prices is assumed. Such a structure would imply a further constraint to the choice problem. In order to retain a simplified model for expositional purposes, technological change is assumed static. Such an assumption could be rationalized if technological change is in fact static, or if expectations of change are myopic. The choice problem can be readily generalized to incorporate either an assumed structure for continuous dynamic technical change (which might affect the effective perennial stock), or discreet shifts in technology such as might occur from the introduction of hybrids. In the latter case, technology specific potential production and capital stock equations (2) and (4) would be introduced and the production functions (1) and (5) generalized to incorporate technology specific outputs.

The stated problem acknowledges alternative uses for limited household labor as leisure and annual or perennial crop production activities. Perennial activities are identified as stock maintenance and adjustment activities, e.g., uprooting, pruning, mulching or new planting, and harvest activities. Crop output is specified as being technologically joint in (1) and (5), implying that productivity of inputs is not completely specific to one crop output and crop production cannot be accurately represented by a series of crop-specific production functions, i.e., production is not non-joint in inputs.

I. Solution of the Dynamic Household Choice Problem

A variety of approaches could be taken to solve the dynamic household choice problem: primal methods employing calculus of variations, or optimal control, or dynamic programming methods focused either on the primal or dual choice problem. Which approach is appropriate depends on the objectives of analysis. A traditional approach of dynamic programming is Bellman's recursive method which recognizes that at the terminal period (or equivalently, at a horizon after which the future becomes economically irrelevant), optimal investment would be zero and a finite closing stock would be expected. In optimal control theory this idea is captured in the form of the transversality condition. Applying this principle, Bellman's method determines optimal choice over time, i.e., the dynamic path of choice, by starting at the terminal period and working backward. At each period, that period's choice problem is constrained by the optimal choices already determined for the future periods. The method is tractable since optimal choice in each period is dependent on the previous period's choice, implying recursive nesting of solutions. A solution derived using this approach is presented below. The approach allows identification of the rules which govern optimal choice and the general form of optimal choice functions.

Conditional upon what functional forms are employed for the objective function, closed-form solutions can be established for the optimal choice functions. However, in general, little is known about the functional form of the objective function and only general forms can be identified. As Weaver (1983) notes, failure to establish closed-form solutions consistent with a given choice problem should not be considered a constraint on learning from empirically-estimated economic models motivated from an explicit theory. Even where closed forms are available, the closed form itself is not an implication of the hypothesized choice problem. Instead, the empirical implications of an economic theory of choice are, in general, independent of the functional form of the objective function. Specification of explicit forms only further restricts empirical implications of the theory and, given the absence of a prior rationale for choosing such explicit forms, potentially obscures our ability to learn from empirical observations. From this perspective, the approach taken here is to employ the theory of choice to identify a set of empirical implications unrestricted by specific functional forms. These empirical implications are then incorporated in the specification of empirical forms of choice functions which are based on functional forms interpretable as approximations to their true forms.

An alternative approach often adopted in motivating empirical models from dynamic optimization problems is to focus on the dual representation of the choice problem. Following this approach, the functional form of the dual function consistent with the primal choice problem is approximated and optimal choice functions are derived using the envelop theorem. The resulting empirical model of choice is consistent with the economic theory of choice and the particular functional form assumed for the dual function. By approximation of the dual function, the choice functions are mutually interdependent, each derived by the envelop theorem from a common dual function. Here, choice functions are directly approximated, since as will be seen, the household choice problem does not imply cross-equation constraints such as symmetry. Future work will explore the implications of this approach through estimation of systems of choice functions derived from an approximated dual function.

To proceed, we simplify the notation by considering the case where x, L, K, Y, Q and C are scalars. Bellman's recursive method, beginning with a terminal condition, provides a solution as follows. Impose the following conditions on (9) at t=T:

(i) $d[L^K(T), K(T)] = 0$

(ii) $I(T) = 0$

(iii) $R(T) = 0$

then,

(iv) $x^o(T) = x^o(T-1)$

(v) $K(T) = D(e_t,T) + K(T-1)$

represent terminal conditions.

By application of these to (11), we have the following first-order conditions:

$$
(12)\quad
\begin{array}{ll}
\text{(i)} & \dfrac{\partial \Psi}{\partial Q_T} = \delta(T)\, U_Q - \lambda_t P_Y(T) = 0 \\[2ex]
\text{(ii)} & \dfrac{\partial \Psi}{\partial C_T} = \delta(T) U_C - \lambda_T\, P_C\,(T) = 0 \\[2ex]
\text{(iii)} & \dfrac{\partial \Psi}{\partial L^Y(T)} = -\, \delta(T) U_L + \lambda_T\, P_Y(T)\, F_{L^Y} = 0 \\[2ex]
\text{(iv)} & \dfrac{\partial \Psi}{\partial L^H(T)} = -\, \delta(T)\, U_L + \lambda_T\, P_X(T)\, J_{L^H} + \lambda_T P_Y(T)\, F_{L^H} = 0
\end{array}
$$

Simplification produces an intuitively interpretable set of choice rules:

$$\frac{U_Q}{U_C} = \frac{P_Y(T)}{P_C(T)}$$

$$\frac{U_Q}{U_L} = \frac{1}{F_{L^Y}}$$

$$\frac{U_Q}{U_L} = \frac{P_Y(T)}{P_X(T)\, J_{L^H} + P_{Y(T)} F_{L^Y}}$$

The latter two imply,

$$P_X\,(T)\, J_{L^H} + P_{Y(T)}\, F_{L^H} = P_{Y(T)} F_{L^Y}$$

which requires that the optimal allocation of labor across enterprises follow the usual equation of marginal value products.

We derive the following general choice functions:

$$(13) \quad Z_T^* = Z\left(\frac{P_X(T)}{P_Y(T)}, \frac{P_C(T)}{P_Y(T)}, \bar{L}(T), e(T), x^o(T-1)\right)$$

where $Z_T^* \equiv (C_T^* \; Q_T^* \; L_T^{Y*} \; L_T^{H*})'$, and * indicates solution values.

Proceeding for further solution, we use Z_T^* and (1) and (5) to determine x_T^* and Y_T^*

$$(14) \quad Y_T^* = F^*(L_t^{H*};\, x^o(T-1))$$

$$= Y\left(\frac{P_{X(T)}}{P_Y(T)}, \frac{P_{C(T)}}{P_Y(T)}, \bar{L},(T)\; e(T);\, x^o(T-1)\right)$$

$$X_T^* = J^*(L_T^{H*};\, x^o(T-1))$$

$$= X^*\left(\frac{P_X\,(T)}{P_Y\,(T)}, \frac{P_C(T)}{P_Y(T)}, \bar{L},(T)\; e(T),\, x^o\,(T-1)\right)$$

Since these solutions are for the terminal period T conditions (11)(i)-(iii) imply the optimal investment decisions are constrained to zero.

To proceed with solution, the terminal period solutions are introduced as constraints on the T-1 period choice problem. In general, allow the t^{th} period solution to be derived through constraints from t+1,...T. For example, at T-1:

$$\Psi(T-1) = \delta(T-1)\big(U(\bar{L}(T-1),\ Q(T-1),\ C(T-1)\big)$$

$$+\ \lambda_{T-1}[P_X(T-1)\ J\big(L^H(T-1);\ x^o_{T-1}\big((L^K(T-1), L^I(T-1), L^R(T-1), x^o(T-2)\big)\big)$$

$$+\ P_Y(T-1)\big(F\big(L^H(T-1), L^Y(T-1);\ x^o_{T-1}\big(L^K(T-1), L^I(T-1), L^R(T-1), x^o(T-2)\big)\big)$$

$$-\ Q(T-1)\big) - P_C(T-1)\ C(T-1)] + \delta(T)\ U\big(\bar{L}(T) - L^*(T),\ Q^*(T),\ C^*(T)\big)$$

The first-order conditions for the T-1 problem can be written:

(15) i) $$\frac{\partial\Psi}{\partial L^H} = -\delta(T-1)U_L + \lambda_{T-1}\big(P_X(T-1)J_{L^H} + P_Y(T-1)F_{L^H}\big) = 0$$

ii) $$\frac{\partial\Psi}{\partial L^K} = -\delta(T-1)U_L + \lambda_{T-1}\big(P_x(T-1)J_{x^o}x^o_{L^K} + P_Y(T-1)F_{x^o}x^o_{L^K}\big) = 0$$

iii) $$\frac{\partial\Psi}{\partial L^I} = -\delta(T-1)U_L + \lambda_{T-1}\big(P_x(T-1)J_{x^o}x^o_{L^I} + P_Y(T-1)F_{x^o}x^o_{L^I}\big) = 0$$

iv) $$\frac{\partial\Psi}{\partial L^R} = -\delta(T-1)U_L + \lambda_{T-1}\left(P_x(T-1)J_{x^o}x^o_{L^R} + P_Y(T-1)F_{x^o}x^o_{L^R}\right) = 0$$

v) $$\frac{\partial\Psi}{\partial L_Y} = \lambda_{T-1}P_Y(T-1)F_{L^Y} - \delta(T-1)U_{L^Y} = 0$$

vi) $$\frac{\partial\Psi}{\partial Q} = \delta(T-1)U_Q - \lambda_{T-1}P_Y(T-1) = 0$$

vii) $$\frac{\partial\Psi}{\partial C} = \delta(T-1)U_C - \lambda_{T-1}P_C(T-1) = 0$$

viii) $$x_{(T-1)} = J\left(L^H(T-1);\ x^o_{(T-2)}\right)$$

ix) $$Y_{(T-1)} = F\left(L^H(T-1), L^Y(T-1);\ x^o_{(T-2)}\right)$$

Simplifying: v) and vi) may be rewritten

$$\frac{U_Q}{U_C} = \frac{P_Y}{P_C}$$

$$\frac{U_Q}{U_{L^Y}} + \frac{P_y}{P_Y F_{L^Y}} = \frac{1}{F_{L^Y}}$$

$$\frac{U_Q}{U_{L^H}} = \frac{P_y}{P_X J_{L^H} + P_Y F_{L^H}}$$

Solution results in:

$$(16) \quad Z^*_{T-1} = Z_{T-1}\left(\frac{P_X(T)}{P_Y(T)}, \frac{P_C(T)}{P_Y(T)}, \bar{L}_T, e(T),\right.$$

$$\left.\frac{P_X(T-1)}{P_Y(T-1)}, \frac{P_C(T)}{P_Y(T-1)}, \bar{L}_{T-1}, e(T-1), x^o(T-2)\right)$$

where $x^o(T-2)$ is defined by (2) and (3).

The conclusion can be drawn that the choice functions can be written for an arbitrary t as:

$$(17) \quad Z^*_t = \left(C^*_t, Q^*_t, L^{Y*}_t, L^{H*}_t, L^{I*}_t, L^{R*}_t, L^{K*}_t, x^*_t, Y^*_t\right)$$

$$= Z\left((P_Y/P_X)_{T,t}\ (P_C/P_X)_{T,t}, \bar{L}_t, e_t, x^o(t-1)\right)$$

where $x^o{}_{(t-1)}$ is defined by (2) and (4) which indicate it will depend on $K(t-1)$ and $x^o(t-2)$ and $(P_Y/P_X)_{T,t}$ $(P_C/P_X)_{T,t}$ represent $(T-t) \times 1$ vectors of expected future prices.

The reduced form choice functions (17) constitute a system which is interrelated due to common dependence of the functional forms of the elements of Z(.) on those of U(.), F(.), and J(.). The choices include both utilization of variable inputs, and adjustment in stocks, e.g. adjustment in tree stock through new plantings I(.) or removals R(.). Before proceeding it is of interest to compare the system of choice functions (17) to those which might follow from a model in which the evolution of price expectations is specified. In the simplest case, where expectations are static $P_{t,t} = P_{\tau,t} \; \forall \; \tau > t$ optimal choices can be written as determined by $P_t = P_{t;t}$. This case is typically assumed in empirical studies of dynamic choice. Alternatively, where a structure is specified for the evolution of expectations, optimal choice becomes determined by the exogenous determinants of that process. For example, if $P_{\tau,t}$ were generated by an extrapolative process such as a non-stochastic adaptive expectation process, $P_{\tau,t} = \lambda P_{\tau-1,t} + (1-\lambda)P_t$, optimal choice could be shown to be determined by λ and P_t. In general, the initial condition of the expectation process, and any exogenous determinants would be determinants of choice. By combination of (17) with any such structure for the evolution of price expectations, a further reduced form representation of choice could be obtained.

IV. VARIATIONS IN STRUCTURE OF THE INTEGRATED MODEL

A. Interrelatedness between Production of Annuals and Perennials

Economists observe choices made by decision makers faced with incentives and technological potential. Choices of production levels of outputs may be interrelated for any of the following reasons: i) preferences are not separable in home consumption of alternative outputs; ii) technology is joint in input utilization; or iii) technology is non-joint in inputs, yet at least one input is not allocatable; or iv) production is non-separable in outputs. Equation (5) specifies a general form for technology which allows annuals and perennials to be joint in production. In a more restrictive case, technology may be non-joint in inputs implying that G(.) may be equivalently represented by a set of output-specific production functions where each output is written as a function of the quantities of each input used in its production. In the case of a profit-maximizing firm, and in the absence of any input constraints, such a technology would imply that for each output, supply is determined only by the output price, not by the prices of other outputs. For the present problem, interrelatedness of production decisions could also follow from preferences U(.) being non-separable in outputs. Preferences specified in (8) assume separability between consumption of annual crop outputs (Q) and perennial crop output. In fact, (8) assumes the marginal utility of consumption of perennial outputs is zero at all levels of consumption. If perennial output were consumed on-farm, that consumption could be introduced to (8) and would imply a reason for interrelatedness in production levels. Non-jointness in inputs where at least one input is not allocatable to specific outputs represents another source of interrelatedness in production choices. A non-allocatable input is defined as one which contributes directly to the productivity of a set of outputs, yet the contribution to one output does not reduce the input's contribution to the other members of the set.

The existence of any of these cases is an empirical issue which must be resolved through knowledge of technical characteristics of production, or examination of other sample evidence. A common implication of each case is that output supplies of annual and perennial crops would be determined by each output's own price, as well as the prices of other outputs. Where none of conditions i)-iii) exist, other crop prices would not affect production decisions of each crop. These results suggest an empirical test for interrelatedness of crops could be based on the simultaneous significance of all crop prices in the system supply functions, or alternatively, the role of other crop prices is found to be significant, see Weaver (1983).

B. Introduction of a Wage Market

Generalization of the integrated model to allow for a wage labor market requires introduction of wage labor L^W in the income-expenditure constraint (7) and the labor constraint (6), and definition of a wage rate W. If the producer is indifferent between alternative uses of labor, the standard result emerges that production decisions (of both annuals and perennials) are recursively separable from the wage market labor supply and household consumption decisions. That is, production decisions can be derived independently of wage market labor supply and household consumption decisions, and (17) could be rewritten as

$$(17') \qquad Z^{**}(t) = Z^{**}\ [(Py/Px)_{T,t},\ (W/Px)_{T,t},\ \bar{L}_t;\ e_t;\ x^o\ (t-1)]$$

where Z** indicates a vector of decisions identical in composition to (17) with the exception of C*. In the case of indifference across alternative employments of household labor, a role for consumer goods prices in determining production decisions depends on the absence of a wage labor market.

An alternative specification of household preferences is important to consider. As Lopez (1984) noted for the case of Canada, farm households may not consider on- and off-farm labor to be perfect substitutes. In this case, production and consumption decisions are by definition not separable, implying that both consumer prices (e.g., P_C) and the wage rate (W) affect choice. This possibility implies that the role of consumer prices and the wage rate is an empirical issue since three distinct theories of household choice exist which suggest different roles for these variables.

For the case of imperfect substitution between on- and off-farm labor, (8) would be redefined to acknowledge differential preferences for place of work, e.g.,

$$U = V\ (L_w,\ L_f,\ Q,\ C)$$

where L_w is wage labor, and L_f is farm labor. By redefinition of the labor constraint (6), the choice problem would be redefined and solution would result in general reduced forms involving consumer prices.

$$(17'') \qquad Z_L\ (t) = Z_L\ (P_y/P_x)_{T,t},\ (W/P_x)_{T,t},\ (PC/P_x)_{T,t},\ \bar{L}_t,\ e_t,\ X^\circ(t-1)$$

where Z_L indicates the vector of choices defined as Z* in (17) with the addition of the labor allocation to the wage market.

If we define (17') as Case 1, and (17'') as Case 2, (17) represents Case 3 in which no labor market exists. The functional forms of Z(.), Z**(.) and Z_L(.) would differ as a result of their derivation from different first-order conditions. However, linear approximations of production decisions for the three cases would be nested since they would differ only by the composition of the vector of independent variables. Specifically, Case 1 and Case 3 would be nested as special cases of Case 2. This nesting provides a tractable means for discriminating among the alternative cases through a series of hypothesis tests.

C. Alternative Approaches to Modelling Subsistence

A subsistence constraint specified as a fixed quantity of a food crop required for consumption would imply that food consumption is not driven by economic concerns generated by objectives, incentives and constraints. In the absence of farm inventories, the existence of such a constraint would imply that either all production is consumed, or when excess exists, it is marketed. This approach has been used extensively in the marketed surplus literature, e.g., Behrman (1966), Haessel (1975), Medani (1975), Hamm r (1986), and Toquero et al (1975). Alternatively, Nakajima (1969) recognized that in the presence of markets the concept of a subsistence constraint can be consistent with preference-based consumption choices only if the constraint is specified as a minimum level of income.

Hammer maintained the assumption that consumption was driven by a fixed consumption target, disallowing a role for food preferences; however, farm inventories were added to facilitate the intertemporal feasibility of achieving the target consumption level. An important issue raised by farm level inventories is the possibility of corner solutions. Hammer's specification assumed excess production could be held in inventory or sold, while deficits were purchased. However, this approach assumes no household cash balance constraint exists, or equivalently, unlimited communal borrowing capacity exists. Further, although farm inventories are often held intra-seasonally, survey results indicate they are rarely held inter-seasonally by the smallholder. These comments suggest the model specified above is preferable for handling subsistence. Specifically, the preference function can be assumed to reflect strong preferences for food prior to the achievement of a subsistence level and greater substitution thereafter.

A further variation of interest for the subsistence case is the existence of a constraint on the availability of market supplies as considered in the rationing literature initiated by Tobin and Houthakker (1950-51), and reconsidered by Pollak (1969), Latham (1980), Howard (1977), and Neary and Roberts (1980). The initial literature in this area considered the effects of rationing on the demand for unrationed goods, establishing that rationing expands demand for unrationed substitutes and reduces that of complements relative to the unrationed case. However, Latham later showed the first effect is indefinite at a theoretical level if substitutability among goods is allowed to differ between the rationed and unrationed case.

Fixed, centrally-determined rations are not observed in Sub-Saharan Africa; however, the availability of products in markets is sometimes uncertain due to transactions bottlenecks or costs. The macroeconomic implications of this issue were considered by Bevan _et al_ (1987). The microeconomic implications of such a prospect might be introduced in (11) for the risk-neutral case through introduction of an expected level of availability of the non-food consumption goods as considered by Bevan _et al_. As suggested by Latham's results, the impact of such rationing on choice levels of unrationed substitute products would be an empirical issue. The Le Chatelier effect would be expected to hold, implying rationing would reduce choice responsiveness to price.

Empirically, the effect of rationing could involve a complication. If cash were available to finance the available ration, the ration level would enter choice functions as would any other exogenous constraint. However, cash balances may fall short of those necessary to purchase available rations. In this case, the cash balance constraint would also be binding, and enter choice functions. This result suggests that simple introduction of rationing into choice functions may fail to capture the complicated choice environment implied by the presence of a ration.

D. Differential Buying and Selling Prices and Parallel Markets

Intra-seasonal price variation and locally-coincident buying and selling patterns would lead to differential prices as assumed by Hammer. Alternatively, differential prices may exist in some cases due to government price fixing and the emergence of secondary, unofficial (or black) markets. Introduction of differential prices of the first type in the model given in (11) is straightforward, requiring only distinction of purchase from selling prices. However, the existence of parallel markets would require introduction of greater transaction costs in the parallel market, or even penalties for being caught (and the risk of being caught) trading in such a market as assumed by Chinn (1978), and Jones and Roemer (1987). If price controls are enforced only for non-food perennial crops, an important distortion could result in food crop production. However, as Jones and Roemer find, the presence and level of penalties as well as the probability of being caught determines the roles of official versus parallel market prices in determining production decisions.

E. Central Government Price Control and Smuggling

The impact of central government price control on producer marketing has been studied by Franco (1981) and May (1985) for the case of Ghana and most recently, in general, by Jones and Roemer. The immediate implication of government price control depends upon the economic efficacy of the controlled price level. That is, when government price P_g exceeds the expected market price net of any penalties imposed on marketing outside of the controlled market, all marketing would go to the government-controlled market. In this case, expected prices would be determined by the controlled price if it is announced during the production planning period, or based on expected government behavior if the controlled price is announced after production plans have been implemented. Alternatively, if the price control fails to exceed the net parallel market price, then expectations would be based on the parallel market. Franco and May employed reduced-form supply functions that reflected these concerns.

Specifically, in the notation of this paper, Franco specified:

$$X_p = X_p(P_x^p - P_x^g)$$

$$X_g = X_g(P_x^g, \Pi_x)$$

$$X = X_p + X_g$$

where the p and g indicate parallel and government-controlled markets, respectively.

May employed a similar specification. Jones and Roemer recognized the possibility of penalties for being caught marketing in the parallel market by introducing both a penalty and a probability of being caught. Importantly, they also recognized the possible discontinuity in choice introduced by the presence of alternative markets. A similar case was considered by Weaver (1982) for U.S. agriculture. The integrated model introduced above can be modified to introduce these possibilities for the case of perennial price control.

The integrated model is restated to include a controlled price P_x^g, a penalty function $f(X_p)$, where $f' > 0$, for marketing X_p in the parallel market at P_x^e, the controlled market delivery X_g, and a probability of being caught $g(X_p)$, where $g' > 0$. The penalty function is equivalent to a tax function: $T(X_p) = g(X_p)\, f(X_p)$. We consider the problem for the case where a labor market exists since presumably off-farm labor opportunities would exist with parallel market functions. The decision problem at period T becomes:

$$\begin{aligned} \Pi_T = {} & P_X^g X_g + P_X^e X_p - T(X_p) + P_Y Y + L^w W \\ & + \lambda_1(\Sigma L + S - \bar{L}) + \lambda_2(X_p + X_g - X) \end{aligned} \tag{18}$$

First order conditions for interior solutions require:

(19) i) $$\frac{\partial \Pi}{\partial L^Y} = P_Y F_{L^Y} + \lambda_1 = 0$$

ii) $$\frac{\partial \Pi}{\partial L^H} = P_Y F_{L_H} + \lambda_1 - \lambda_2 J_{L_H} = 0$$

iii) $$\frac{\partial \Pi}{\partial L^w} = W + \lambda_1 = 0$$

iv) $$\frac{\partial \Pi}{\partial X_g} = P_x{}^g + \lambda_2 = 0$$

v) $$\frac{\partial \Pi}{\partial X_p} = P_X{}^e - T_{X_p} + \lambda_2 = 0$$

vi) $$\frac{\partial \Pi}{\partial \lambda_1} = \Sigma L + S - \bar{L} = 0$$

vii) $$\frac{\partial \Pi}{\partial \lambda_2} = X_p + X_g - X = 0$$

Solution of (19) i) - iii) clearly requires $T(X_p) = g(X_p)\ f(X_p)$ to be convex in X_p. However, whether such a condition exists depends completely on the government's design. We assume $T''(X_p) < 0$ which would be in the government's interest to reduce X_p. More important is the possibility of corner solutions introduced by government price control.

From (18), for the case of corner solutions

If $X_p = 0, \quad P_x^e - T_{x_p} + \lambda_2 \leq 0, \quad \lambda_2 = -P_x^g$

(20) i) $X_g^* = X^*$ where X^* is determined by P_x^g

If $X_g = 0, \quad P_x^g + \lambda_2 \leq 0, \quad \lambda_2 = -(P_x^e - T_{x_p})$

ii) $X_p^* = X^*$ where X^* is determined by $P_x^e - T_{xp}$

iii) If an interior solution occurs,

Y and X can be written as functions of either P_x^g or $P_x^e - T_{x_p}$ since they are equivalent.

This follows from iv) and v) which are equalities when an interior solution occurs, implying:

$$P_x^g = P_x^e - T_{x_p}$$

Further, in this case i) - iii) can be seen to be separable from the marketing decisions defined by iv) and v). That is, iv), v) and vii) are sufficient to determine X_p and X_g given solution of the remaining first-order conditions which determine L^Y, L^H, S, X, and Y.

The implications of these results are that i) production and marketing decisions are separable and ii) production depends on the controlled price $P_x{}^g$, or equivalently, $P_x{}^e - T_{x_p}$; a result that differs from Franco's and May's specification. Further, as noted by Jones and Roemer, when an interior solution occurs, $X^*(P_x{}^g,\ldots) = X^o(P_x{}^g,\ldots)$ where $X^o(.)$ solves the choice problem for the case where an open market exists, i.e., no parallel market exists. It is important to note, however, that $X^*(P_x{}^g,\ldots) \neq X^o(P_x{}^o,\ldots)$ where $P_x{}^o$ is the competitive open market price. In fact, to clarify Jones and Roemer's claim, where $P_x{}^g < P_x{}^o$ it follows that $X^o(P_x{}^o,\ldots) < X^*(P_x{}^g,\ldots)$.

Where an interior solution does not occur, we see from viii) and ix) that two cases occur. Where viii) holds and $X_p = 0$, $P_x{}^g$ determines production decisions, while when ix) holds and $X_p = X$, $P_x{}^e$ determines production decisions. As also noted by Jones and Roemer, when $X_p = X$, $X^*(P_x{}^e) < X^o(P_x{}^e)$, a direct result of the penalty for being caught.

The implications of price control for model specification follow from the above generalization of the integrated model. The most important of these implications is that the functional form of the choice function as well as its arguments depends upon whether

1) $X_p = 0$, 2) $X_g = 0$, or 3) $X_p \neq 0$, $X_g \neq 0$.

Over time an individual could be expected to find each of the three alternative cases optimal. At any given time, differences in preferences across individuals would similarly result in different cases being observed. This result implies that a model of an aggregate of individuals must acknowledge these different response possibilities and model both the i) choice function for particular response cases, and ii) the probability of particular responses. For example, an aggregate choice function might be written:

$$(21) \qquad Z = \phi\, X^{2}(P^{g}_{X}\ , w,\ P_{Y}) + (1-\phi)\ \lambda^{P}{}_{X}(P^{e}\ , w, {}_{Y}P\ )$$

where ϕ is the probability of $X_g \neq 0$.

V. EMPIRICAL IMPLEMENTATION

A. Elements of Model Specification

The approach taken here is to base model specification on observed economic systems and the implications of an economic theory of choice which is consistent with those systems. Critical issues which must be resolved include: i) identification of inputs and outputs that are variable in the production period; ii) identification of input flows and household characteristics which are fixed in the production period; iii) specification of the incentives and constraints faced by the household, including the set of production alternatives faced; and iv) specification of the general determinants of household welfare as represented by household preferences. Resolution of each of these issues must be conditional on the sample analyzed. When dealing with data representing an aggregation of households, the accuracy of any generalization cannot be exact. The set of alternative crops which may be grown may vary significantly across regions due to variation in soils, climate and available production technology. Where such variation exists, regional disaggregation of data is appropriate subject to data availability. Further, extreme variation in household preferences may exist across ethnic groups, over scale of agricultural activities, or over characteristics of technology, e.g., irrigation. Again, where such variation exists, or is suspected, use of data disaggregated or stratified according to such subsets would be preferred.

Similar heterogeneity within a sample may result in time series observations taken from periods over which technology, constraints or preferences abruptly change.

B. Alternative Approaches to Measurement of Price Expectations

Implementation of the dynamic reduced form presented in (17) requires the measurement of a schedule of future expected prices from the current period to the planning horizon. The objective of any approach is to specify a systematic structure to the time path of expectations which will allow representation of the series of future expectations by a small set of indicators. A traditional approach is to assume expectations are static, allowing the sequence of future expectations to be represented by the expectation for the current period. While this approach will be adopted in the further discussions of empirical implementation, it is of interest to explore alternatives available for relaxing this assumption. A variety of approaches have been taken in the past to model this temporal schedule of expectations. These will be reviewed in the following subsection. However, before proceeding it is of interest to establish approaches suggested by economic theory.

Three possibilities exist. First, the theory of functional separability identifies the conditions under which a reduced-form choice function of a vector of prices can be equivalently represented by a reduced-form choice function of a vector of indexes. This approach would allow representation of the schedules of future price expectations by univariate indexes. A second approach is estimation of partial reduced forms drawn from the first-order conditions (15). In this case, the endogenous variable left as an independent variable is a function of the expectation vector, and can therefore serve as a summary statistic. The third approach relies upon the rational expectations hypothesis which provides the basis for endogenizing expectations. This approach simply translates the problem of measuring the expectations schedule into one of measuring the exogenous information set available in the current period upon which the schedule of expectations are based.

The application of functional separability to the current problem requires that the Lagrangian in (14) be weakly separable in choice variables over the time horizon. That is, the vector of choices

$$[Q(T), Q(T-1)\ldots Q(t), C(T), C(T-1)\ldots C(t),$$
$$L^H(T),\ldots L^H(t), L^K(T),\ldots L^K(t),$$
$$L^I(T),\ldots L^I(t), L^R(T),\ldots L^R(t), X(T),\ldots X(t)]$$

must be separable according to the partition of subgroups (indicated by [ ]):

$$\big([Q(T),\ldots Q(t)], [C(T),\ldots C(t)], [L^H(T),\ldots L^H(t)], [L^K(T),\ldots L^K(t)],$$
$$[L^I(T),\ldots L^I(t)], [L^R(T),\ldots L^R(t)], [X(T),\ldots X(t)]\big)$$

The sufficient conditions for such weak separability are that the marginal rates of substitution between like choice variables at different times be independent of other choice variables, e.g.,

$$\frac{\partial \dfrac{\partial\Psi}{\partial Q(\tau)} \Big/ \dfrac{\partial\Psi}{\partial Q(\tau-1)}}{\partial\Psi/\partial Z(\tau-j)} = 0 \qquad \forall\tau, i, \text{ and } j,\ Z(\tau-j) \neq Q(\tau-j)$$

Although by definition the utility function is temporally separable, at any time τ it could not, in general, be expected to be separable across choices. Further, the perennial production function (1) is by definition not separable across either contemporaneous or differently dated choices, e.g. $L^H(t)$ and $x^o(t,v)$ since $x^o(t,v) = d(L^K(t,v), K(t,v)) + x^o(t-1,v)$. This result implies a similar conclusion for annual production (5). The conclusion must be drawn that temporal choice separability represents an inappropriate restriction on the functional form of the Lagrangian for the current application and should not be adopted as a means of simplifying the measurement of the temporal schedule of price expectations.

The alternative of developing an empirical model based on partial reduced forms of (12) could proceed by solution of 12 i) - 12 v) for L^*, then vi) - ix) would define partial reduced forms for the vector $Z^\circ = (Q^\circ, C^\circ, X^\circ, Y^\circ)$ where $Z^\circ = Z^\circ(L^*, P_c, P_y)$. However, ultimately this approach would lead to the same requirement of measuring a temporal schedule of price expectations as was encountered in the complete reduced-form representation. The only approach which would truly eliminate all prices would be use of viii) - ix) with L^H as an endogenous explanatory variable. In this case, an instrument vector could be used containing available exogenous information on future prices. While tractable, this approach could only converge to the efficiency of a full reduced-form approach as the instrument vector approached incorporation of the complete schedule of price expectations.

The third alternative to consider follows from adoption of a theory of expectation formation which explicitly endogenizes the expectation schedule. If the expectation theory is based on currently available information, then each expectation in the time schedule would be a function of the same information set. The rational expectations hypothesis provides a basis for this approach. By adding a demand function for each product involved, the price is endogenized as a function of demand and supply determinants, e.g., in the present case the price vector might be written as

(22) $$P_\tau = P(\phi_\tau, X^o_{\tau-1}, T_\tau) + \varepsilon_\tau$$

$$\text{and} \quad P^e_\tau = E(P_\tau) = P_\tau(\phi_\tau, X^o_{\tau-1}, T_\tau) \; V \; \tau \; t, \ldots T$$

where $P_\tau \equiv (P_X(\tau), P_Y(\tau), P_C(\tau))$

ϕ_t is a vector of demand determinants for $X_t \ldots X_\tau$, $Y_t, \ldots Y_\tau$

By substitution into (17), we have a reduced form

(23) $$Z^*_t = Z(P^e_t, P^e_{t+1}, \ldots P^e_T, T_t, e_t, X^o_{t-1}).$$

However, since all expectations are based on the common set of information available at time t, here represented by (ϕ_t, X^o_{t-1}, T_t), use of the composite function theorem would allow (23) to be written:

$$Z^*_t = Z\ (\phi_t,\ X^o_{t-1}, T_t,\ e_t)$$

The problem with this approach, of course, is the loss of identification for the comparative-static effects of changes in individual expected prices. Alternatively, such comparative-static effects could be interpreted as of little interest. Of greater relevance is the effect of changes in information which affect the time path or sequence of prices. From this perspective, of interest is:

$$\partial Z^*_{it}/\partial\phi_t = \sum_{\tau=1}^{T} (\partial Z^*_{it}\ /\partial P^e_{t+\tau})\ \partial P^e_{t+\tau}/\partial\phi_t$$

Although $\partial Z^*_{it}/\partial P^e_{t+\tau}$ could not be estimated, estimates of $\partial Z^*_{it}/\partial\phi_t$ may arguably be of greater interest since price expectations are unobservable. This approach is typically taken in optimal control and dynamic programming approaches to solution of problems of dynamic choice.

A final approach is to employ only past prices as elements in the information set ϕ_t. Following Weaver (1982), the efficient price expectation based on past prices would be the optimal extrapolation of prices given by:

(24) $$P^e_t = \phi(\beta)P_t$$

Here, for any $\tau > t$,

$$P_\tau^e = \phi_o + \sum_{i=1}^{\tau-t}\phi_i P_{\tau-i}^e + \sum_{j=\tau-t-1}^{-\infty}\phi_j P_{\tau-j} = \phi(\beta)P_\tau^e$$

or by substitution,

$$P_\tau^e = \pi^\tau(B)\ P_t$$

since $P_{t-i}^e = P_{t-i}$ for all $i < \tau-t$.

By estimation of the optimally extrapolative filter $\phi(\beta)$ all future expectations can be forecast. For any future period τ the filter $\pi^\tau(B)$ is defined by the parameters of $\phi(B)$. Although forecasts will, in general, be highly correlated the use of this type of forecast as a measure of expected future prices is tractable. A further problem which must be resolved in implementing any model incorporating a non-static profile of future expectations is specification of the terminal point T.

C. Measurement of Potential Perennial Crop Production

A further measurement issue which must be resolved is that of $x^o(t-1)$. In structural form, $x^o(t-1)$ was specified in (2) as a measure of potential output which depends upon current adjustment through investment of labor and tree stock. A number of approaches could be taken to model this unobservable. Akiyama and Trivedi (1987) employ the vintage capital method in which an estimated yield-age distribution is used with an estimated tree age profile to calculate the output potential at any time t. Changes in tree stock are estimated and used to adjust the tree age profile through the sample. The estimated yield-age profile is typically held constant throughout the sample, despite probable responsiveness of yields to labor and weather events. Within this context, the fixed yield profile can be interpreted as a fixed expected yield profile. However, even this interpretation would suggest expected yields do not respond to current and planned actions. Given this assumption, expected production would only vary with application of harvest labor. Further, this approach does not allow for the production potential estimate to vary over time as a result of past experience and anticipated events.

An alternative approach is to interpret production potential as an unobservable that is predicted by the producer depending upon past experience and events preceding the application of inputs during the current growing season, e.g. L^K, L^I, or L^R. Again, a useful means of making this prediction is to efficiently utilize all available past history of actual production in an optimal extrapolation, e.g.,

$$X^o(t) = \theta(B)\ X\ (t)$$

D. Past Approaches to Modelling Perennial Supply

Akiyama and Trivedi, Hartley *et al.*, Wickens and Greenfield, and Dowling represent recent examples of perennial models which confront the problem of estimation of perennial production choices. In each case, only a single crop is considered. Since the approaches are closely related, a general summary will be presented. Placing the approach taken in these papers in the current notation, dynamic decisions lead to planned production given by substitution of solutions to 15 i) - 15 vii) into viii):

$$(25) \qquad X^{*}(t) = J\left(L^{H^{*}}(t-1),\ X^{o}_{t-1}\right)$$

Past approaches have defined $X^{*}(t)$ as planned production as distinguished from actual production X(t). The two are typically related by the definitional:

$$(26) \qquad X(t) \equiv X^{*}(t) \left\{ \frac{X(t)}{X^{*}(t)} \right\}$$

where $[X(t)/X^{*}(t)]$ is interpreted as contemporaneous adaptation to deviation of current prices from expected, and modeled as (e.g., in Akiyama and Trivedi):

$$(27) \qquad \frac{X(t)}{X^{*}(t)} = f\left(P(t)/P^{e}(t)\right) U(t)$$

where U(t) is a stochastic shock. $X^{*}(t)$ in (26) is modeled by Trivedi, and Akiyama and Trivedi as

$$(28) \qquad X^{*}(t) = X^{o}(t) \left\{\frac{X^{*}(t)}{X^{o}(t)}\right\}$$

where $X^{o}(t)$ is modeled as dependent on past investment decisions, e.g.,

$$X^o(t) = k_1 X^f(t)^\varepsilon \quad \text{where}$$

(29) $$X^f(t) = \sum_v \delta(t,v)\, K(t,v)$$

that is, feasible production $X^f(t)$ is determined by a vintage distribution of yields and plantings. To complete the model, $[X^*(t)/X^o(t)]$ is interpreted as the deviation of planned from feasible production and is explained as a function of a distributed lag of past prices; e.g.,

(30) $$\left\{\frac{X^*(t)}{X^o(t)}\right\} = k_2 \prod_{i=0}^{m} P(t-i)^{\beta i} \qquad \beta i \geq 0$$

A motivation for (28) might follow from homotheticity of J(.) which would allow it to be written:

(31) $$X^*(t) = H\,[L^{*H}(t-1)]\, X^o(t-1)$$

however, from (17) it is apparent that $L^{*H}(.)$ would be determined by the temporal schedule of prices.

From (31) it is also clear the approach is one of estimating a partial reduced form in which endogenous independent variables are further modeled in reduced forms. If the approach is to be consistent with the theoretical model, the specification in (30) must be interpreted as an index of future expected prices measured by a distributed lag of past prices. Such an interpretation would require temporal choice separability as defined above, an assumption already implicit in the single-crop approach taken; however, this assumption is relaxed in the model presented here.

The conclusion can be drawn that past approaches have expanded the model of the perennial production function beyond that presented in 15 viii) to introduce (i) a distinction between planned and actual production, and (ii) technologically feasible and planned production. These distinctions are supported by the assumption that a recursive ordering exists in which actual production is conditional on planned, and planned on feasible production.

While (25) defines a structural basis for conditionality of planned on feasible production X^{o}, the only basis that exists for the conditionality of actual on planned output is one of temporal ordering and the existence of disturbance and possible reaction to disturbance during the time separating plan formation and realization of production. If $X^{*}(t)$ in (26) is interpreted as a solution of a dynamic optimization problem at a planning date $t-\tau$ [i.e., X^{*} is properly written $X^{*}(t)_{t-\tau}$], then $[X(t)/X^{*}(t)_{t-\tau}]$ would be interpreted as adaptation of the plan to changes in information. Although this interpretation is intuitively appealing it does presume that at t when new information occurs, the decision maker ignores the dynamic optimization problem (which would lead to $X^{*}(t)$ properly defined and achieved despite past plans from $X^{*}(t-\tau)$, and instead simply adapts $X^{*}(t-\tau)$. This is the assumption employed by Trivedi. Expected prices $P^{e}(t-\tau)$ are assumed to lead to dynamically optimal plans $X^{*}(t)_{t-\tau}$. However, when random shocks to expected prices occur at t, Trivedi assumes the decisionmaker "adapts" according to

(32)
$$\frac{X(t)}{X^*(t)} = f(\varepsilon_t)\ U_t$$
$$\varepsilon_t = \frac{P(t)}{P^e(t)}\ .$$

Why the producer "adapts" to ε_t rather than resolving his dynamic optimization problem to determine the new optimal action is unexplained. However, such adaptation would be consistent with a theory of dynamic optimization only if, in the short run during which adaptation occurs, resources are fixed relative to their status at other decision times.

In summary, the Trivedi model structure can be written in the form of (15) as:

(33)
$$X(t) = X^*(t)\ \left\{\frac{X(t)}{X^*(t)}\right\}$$
$$= X^*(t)v_t$$

where

(34) $$v_t = f(\varepsilon_t)u_t \qquad \varepsilon_t = P(t)/P^e(t)$$

(35) $$X^*(t) = J[L^{*H}(t);\ X^o(t-1)]$$

where $X^o(t-1)$ is potential production estimated from the expected yield distribution and tree stock history, and $L^{*H}(t)$ follows from (16):

$$(36) \qquad L^{*H}(t) = L^{*H}[P_Y/P_X)_{T,t}, (P_C/P_X)_{T,t}, T, e_{t-1}, X^o(t-1)]$$

If we assume J(.) is homothetic, then (27) can be placed in context of Trivedi:

$$(37) \qquad X^*(t) = H[L^{*H}(t)]X^o{}_{(t-1)}$$

$$= h[(P_Y/P_X)_{T,t}\ (P_C/P_X)_{T,t}, T, \varepsilon_t, X^o(t-1)]\ X^o{}_{(t-1)}$$

and it can be seen that Trivedi's specification (30) is a specialization of h(.). From this interpretation, Trivedi appears to have represented the vectors of price expectations, e.g., $(P_y/P_x)_{T,t}$ as determined by and represented by the geometric function of past prices given in (30).

E. An Econometric Approach to Estimation of the Integrated Model

The general dynamic reduced form of the integrated model developed above (17) may be summarized as:

$$(38) \qquad Z_t^* = Z[(P_Y{}^e)_{T,t}, (P_X{}^e)_{T,t}, (P_C{}^e)_{T,t}, \bar{L}_t, e_t; x^o(t-1)]$$

where

$(P_z^{\,e})_{T,t}$ is a vector of expected prices over the horizon $t,\ldots T$

$Z = Y,X,C,L.$

As indicated in the previous sections, measurement models can be used to model the unobservable price expectations, e.g., from (24):

$$(39) \qquad P_Y^{\,e}(t+\tau,t) = \phi_Y(B)P_Y^{\,e}(t+\tau,t)$$

where $B^{\tau-i}\, P_Y^{\,e}\, (t+\tau,t) = P_Y(t-i)$ for all $i > 0$.

Similarly, a measurement model was introduced for the previous period's production potential $x^o(t-1)$, from above:

$$(40) \qquad x^o(t-1) = \Theta(B)\, x(t-1)$$

In order to estimate the model composed of (38) - (40), a two-step procedure is proposed. First, optimal extrapolations are chosen using ARIMA estimates of $\phi_Y(B)$ and $\Theta(B)$, and series of estimates are constructed for $P_Y^{\,e}$, $P_X^{\,e}$, and $x^o(t-1)$. The resulting series are employed to estimate (38) as a seemingly unrelated system. Contemporaneous correlation in the error structure of (38) is expected to result from stochastic shocks to the first-order conditions.

The functional form of (38) is not suggested by the theory of choice. Although specification decisions could be made which might lead to particular forms (see, e.g., Akiyama and Trivedi where multiplicative definitions are introduced which appear to motivate log-linear forms), the choice of functional forms is ultimately an empirical issue. A variety of approaches are possible as reviewed by Weaver (1982); however, for Sub-Saharan data sets, parsimony in parameterization is required due to limited degrees of freedom. This constraint supports the strategy of limiting the search to linear or log-linear forms.

A further issue of some concern is the joint dependency of relative expected prices on common determinants. Although simultaneity among production levels and production incentives does not occur when expected prices are employed, expected prices may be jointly dependent upon factors which are common across commodities such as those which determine the demand for food. Where this occurs, the joint dependency would lead to high correlation among relative expected prices. For example, in abbreviated notation, consider the case of a model of the form

$$Z_t = Z(P^e_{T,t}, \phi_t) + \varepsilon_t$$

Suppose the vector of expected prices are jointly dependent on factors affecting household demand, e.g., δ. The vector δ might even include elements of ϕ when consumption and production decisions are not separable. In any case, the expectations could be written as:

$$P^e_{T,t} = P_T^e\,(\delta_t,\ \gamma_t) + V_t$$

where γ_t is a vector of other determinants of the expectation.

Under these conditions, $P^e_{T,t}$ remains exogenous to Z_t, implying simultaneous equation methods would not be required to achieve consistent estimators unless V_t and ε_t are specified as correlated due to joint dependence of Z_t and $P^e_{T,t}$. Because such joint dependency is not hypothesized, the sets of equations (39) and (40) can be estimated independently.

Estimation of the model composed of (38) -(40) may proceed with the addition of two further relationships motivated from the assumption that expectations and potential production are unbiased estimates of actual prices and production, respectively. For example,

(41) $$P_Y(t+\Gamma) = P_Y^e\,(t+\Gamma,t) + V_t,\ V_t \sim N(0,\sigma_v^2)$$

and for $X°(t-1)$:

(42) $$X(t) = X°(t-1) + U_t, \quad U_t \sim N(0,\ \sigma_u^2)$$

If a linear form is adopted for (38) and $Z_t = Z^*_t + \varepsilon_t$, then so long as ε, V_t and U_t and U_f are independent, forms implied by (39) and (41), and (40) and

(42) can be identified and estimated as independent ARIMA models. Following this first step, the ARIMA models can be used to generate predicted values which from (43) and (42) are interpretable as $P_y^e(.)$ and $X°(.)$. Use of ARIMA representations of the AR filters [e.g., $\phi_y(B)$ Θ (B)] allow for an increase in the efficiency of predicted values due to the parsimony of ARIMA vs. AR representations. For the same reasons, the ARIMA approach allows the researcher to avoid arbitrary truncation of an AR filter.

In the second step of the estimation strategy, predicted values for $P_y^e(.)$ and $X°(.)$ are employed to estimate a system of reduced form choices implied by (38). Because the stochastic errors of such a system are likely to have been generated by a common set of stochastic disturbances in the first-order conditions, the system's error structure is assumed to be spherical. This stochastic interrelatedness implies a gain in efficiency in estimates can be achieved through estimation of the system using Zellner's efficient estimator, an application of Aiken's generalized least squares.

The existence of alternative roles of labor markets suggested by Cases 1-3 as represented by equations (17), (17') and (17'') suggests the need for empirical consideration. As noted above, production decisions for Case (1) and Case (3) are nested under Case (2) in the case where equations (17), (17') and (17'') are approximated by functional forms that are linear in parameters. In this case, standard joint hypothesis tests can be employed to discriminate among the alternative models.

VI. CONCLUSIONS

The objective of this paper has been to develop a general theory of agricultural production decisions, or more simply crop supply, from a micro-economic model which is consistent with the general characteristics of agricultural economic systems observed in Sub-Saharan Africa. A review of the Sub-Saharan Africa systems identified the following general characteristics with which a model of crop supply should be consistent: i) multiple outputs are produced; ii) both annuals and perennials are often produced by the same household; iii) labor represents the dominant input and also represents a limiting factor of production; iv) household consumption and production decisions may be interrelated due to absence of labor market opportunities or imperfect substitutibility of household labor employment on-and-off-farm; v) product markets may be regulated, leading to the evolution of parallel markets; vi) households may consume some production directly; and vii) on-farm inventories are most often non-existent.

The theory of choice developed is labelled an integrated model because of the integration of annual and perennial crop choice. Past theoretic and empirical models have assumed production of annuals and perennials to be separable or independent in choice. The theory presented results in choice functions which are derived in general form from the necessary conditions for optimization. Because comparative statics of this choice problem are, in general, indeterminant, their signs remain an empirical issue. The theory of choice is generalized to accommodate differing roles of labor markets and the existence of government price control and parallel markets. Issues involved in empirical implementation of integrated models are evaluated including measurement of i) price expectations, and ii) potential production of perennials.

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